Endogenous Incentive Mechanism behind Online Review Writing

Shilpi Mukherjee[*]
Sukanta Bhattacharya [†]
and
Anirban Mukherjee [‡]

October 2020

Abstract

Online retail markets have exploded in recent times with the growth in technology. Most of these online stores marked by the presence of a review writing forum. A lot of empirical work has been done on the efficacy or validity of such reviews. In this paper we model an endogenous incentive mechanism to help us understand as to why people write online reviews at a personal cost, without receiving any explicit benefit. We use the pivotal argument used in the Information Aggregation mechanism in the voting literature to model the review writing process. We are able to show review writing emerge as an equilibrium strategy at small positive costs of review writing even when there are no extraneous benefits associated with review writing.

JEL Classification Codes: C7, D7, D86

[*]Clemson University. Email:shilpionweb@gmail.com
[†]University of Calcutta. Email:bsukanta@gmail.com
[‡]University of Calcutta. Email:anirban1976@gmail.com

1 Introduction

Any contract embeds a potential prisoner's dilemma situation which, in absence of any enforcement agency, leads to an equilibrium where both parties cheat. The theoretical modelling of contractual exchange, for the most part of economics literature, implicitly assumes the existence of a state-like enforcement agency which can detect and punish the one who deviates from the contractual agreements. In absence of a strong state, the only way to avoid the non cooperative equilibrium is to play the prisoner's dilemma game for infinitely many times. However, it is unrealistic to assume that two persons are having the same transactions over and over again for infinity. A more plausible interpretation of the infinitely repeated game between two individuals can be that two individuals are matched and have played the game for finite times but their history becomes common knowledge within their community. Communities or a similar organisation, by acting as the reservoir of information and cultivating a norm of social sanction against the cheater can ensure the cooperative equilibrium (Kandori, 1992; Greif, 1993; Dixit, 2003; Milgrom et al., 1990).

The key requirement for the reputation based mechanism to work is information sharing. Historically, communities have been promoting the norm of communal gathering, usually by fixing a day to visit religious site (e.g. Sundays for Church, Fridays for Mosque, Thursdays for some Hindu Temples), which facilitates free exchange of information. The advent of information technology radically remodelled the costs of information sharing as well as the structure of community. In a world, characterised by unprecedented mobility, traditional communities are breaking down. The breaking down of traditional community has a pitfall for the reputation mechanism. The existing literature on reputation mechanism does not explicitly model the costs of information sharing for individual agents. This is however, not a critical omission in the traditional set up where community organisations actively promoted the norms of information and the costs of not complying with such norms is far greater than the costs of information sharing. The costs of information transmission is also present in case of formal, legal cases where such costs takes the form of costs of writing legal documents, lawyer's fee etc. However unlike reputation where information is disseminated in a decentralized way, in legal cases such information is deposited to a central body. From the perspective of litigant, a promise of binding legal solution in exchange of that money, justify incurring those costs. However, in legal cases, just like informal reputation mechanism, there are people who do not directly benefit from the verdict of the court cases but nonetheless disseminate information – they are called witness. Testifying in courts also involve costs such as costs of travelling or lost wage, but once subpoenaed, a witness is legally obligated to do so.

In this paper, we examine the motivation behind writing product reviews on internet which is another form of information dissemination. But unlike legal case or community based reputation mechanism there is no costs for not disseminating information on internet. Nevertheless, people undertake the work of review writing even though they are fully aware that whatever wrong being done to them cannot be reversed and there is always a possibility of free riding (Resnick and Zeckhauser, 2002). In this context, it is an intellectual challenge to look into the rationale behind review writing.

Even though our research focuses on a relatively narrow area of review writing for products on the internet forums, the implications of our endeavour are much wider as virtual reputation for individuals is becoming important as well. Recent developments around the allegations of sexual harassments posted on social media – more commonly referred to as the "me too" movement – which resulted into the social sanctions against the accused perpetrators show the criticality of internet reputation. Inspite of the growing importance of internet reputation, the literature around this issue is rather thin. A more general trend of this literature is to look at the effect of online reviews on the reputation of the sellers or purchase decisions by the buyers (Cabral and Hortacsu, 2010; Lee et al., 2008; Duan et al., 2008). However, unlike the community set up where reputation information can be verified, one problem of internet reputation is the possibility of cheap talk which has an important bearing on the reliability of internet reputation. In a paper related to this area Hu et al. (2011) try to check empirically if reviews are manipulated and show that manipulation strategy of the firm is a monotonically decreasing function of product's true quality and the mean consumer rating.

We have come across a few papers that lists the factors that could be important motivations behind review writing on internet. According to Dellarocas et al. (2010) such motivation includes a myriad of factors such as product involvement (venting out extreme positive or negative feelings about the product), other involvement (concern for others possibly motivated by altruism), message involvement, social benefits arising due to participating in social word of mouth (WOM) or economic benefits provided by the producers. In another paper, Jøsang et al. (2007) discuses the importance of reviews in internet market place as the consumer has no way to test the physical quality of the product – *squeeze the orange*, so to say, she has to rely on customer rating to gauge the quality of the product. The author recognises the importance of having a theory of review writing. Our paper fills in the gap by providing a theory which discusses a rationale behind writing product review on internet.

We borrow our theoretical structure from the voting literature where a self seeking person's motivation to vote remains a puzzle. Among several explanations put forwarded for voting behaviour, we draw heavily on pivotal voting theory expounded by Feddersen and Pesendorfer (1996, 1997). The idea is that before deciding to vote, a voter tries to calculate her expected utility of voting. She only votes if she thinks that the act of voting by her can tip the scale in favour of her preferred candidate which was otherwise weighing in on the other person. Thus the person undertakes the cost of going to the polling booth, casting the vote which is costly to her only if she expects that her marginal vote has the power to change the outcome of the election. We employ a similar argument in our paper to delineate the motivation behind writing the review.

2 Model Structure

The construction of the model is as follows

$\pi p + (1-\pi)(1-p)$ is the expected utility that a consumer receives from the monopolist in the beginning of the first period. Let μ be the posterior probability that a consumer forms based upon the number of good reviews and bad reviews that a consumer sees. Let us assume that there are x good reviews and y bad reviews. Thus the posterior probability that the monopolist is good or bad is a function of x, y, and the parameter value of p and π. The consumer will not purchase the commodity as long as

Expected utility from purchasing the commodity after updating the probability
$<$ ***Expected utility from purchasing a commodity from a new entrant***

Since, the consumer does not have any information about the new entrant, she will apply the same prior probabilities for the new entrant as she had applied for the original monopolist.

Now for the application of the pivotal argument,let us suppose that the consumer has received a good quality signal. It is now worth examining under what conditions will she write the review. As has already been argued earlier she will only think of writing the review, if she considers herself to be pivotal, based on whatever information is available to her. Also, we have assumed that any monopolist needs a certain amount of market demand to sustain herself in the market, without which she is by default obliterated from the market. The consumer knows this and she also knows that if the posterior probability of the monopolist being honest is such that the expected utility that the consumer expects to get after seeing x good reviews, and y bad reviews is less than the expected utility from a new entrant, nobody else will purchase from this monopolist , and the monopolist will be driven out. The consumer who had received the good signal has an incentive to keep the monopolist in the market, she only decides to convey her signal for which she has to incur a cost, if by writing the review, she can increase the posterior probability to the extent that it is now greater than the expected utility from the new entrant.

Thus the condition in which the monopolist will be driven out of the market is

$$\mu(x,y)p + (1-\mu(x,y))(1-p) < \pi p + (1-\pi)(1-p)$$

$$\Longrightarrow \mu(x,y) < \pi$$

and will stay in the market if

$$\mu(x,y)p + (1-\mu(x,y))(1-p) \geq \pi p + (1-\pi)(1-p)$$

$$\Longrightarrow \mu(x,y) \geq \pi$$

The consumer who has received a good signal, henceforth denoted by g will write the review if

$$\mu(x,y)p + (1-\mu(x,y))(1-p) < \pi p + (1-\pi)(1-p)$$

$$\Longrightarrow \mu(x,y) < \pi$$

and

$$\mu(x,y,g)p + (1-\mu(x,y,g))(1-p) > \pi p + (1-\pi)(1-p)$$

$$\Longrightarrow \mu(x,y,g) \geq \pi$$

$\mu(x, y)$ can be calculated by Bayes' Rule where

$$\begin{aligned}
\mu(x,y) &= \Pr(H/x,y) \\
&= \frac{\Pr(x,y/H).\Pr(H)}{\Pr(x,y/H).\Pr(H)+\Pr(x,y/O).\Pr(O)} \\
&= \frac{\pi p^x(1-p)^y}{\pi p^x(1-p)^y+(1-\pi)p^y(1-p)^x} \\
&= \frac{1}{1+\frac{(1-\pi)}{\pi}\left[\frac{(1-p)}{p}\right]^{x-y}}
\end{aligned}$$

Since the consumer who has got the good signal, can contribute only one review, and x and y are both whole numbers, it follows that $\mu(x, y) = \pi$ if $x = y$. Thus the consumer who has got a good review is only pivotal if she observes x good reviews and y bad reviews, where $x = y - 1$, so that one good review by the consumer can make $\mu = \pi$

The exact opposite happens when the consumer uses a bad commodity or gets a bad signal. She wants to drive the monopolist out of the market, which can only happen if the updated probability is such that $\mu < \pi$. Here ,the consumer who receives a bad signal considers herself to be pivotal if she observes equal number of bad and good reviews, because when $x = y$, $\mu = \pi$

Therefore the condition under which the consumer considers herself to be pivotal is if she observes equal number of good and bad reviews before deciding on whether she wants to write a review herself which he/she bases on the condition that the gain in utility from review writing is greater than gain in utility from not writing.

We model this in the following way. Let us denote by μ_s^i the probability that an agent is pivotal after signal $i \in \{g, b\}$ in state $s \in \{G, B\}$. Notice that after getting the signal g, an agent, with a cost of writing review equal to c writes a review with positive probability if and only if

Expected gain in utility from keeping the monopolist $>$ *Expected gain in utility from a* *new entrant , who will enter if the monopolist leaves due to insufficient demand.*

$$\Pr\left(G|piv^{g}.,g\right)p+\Pr\left(B|piv^{g}.,g\right)(1-p)-c\geq\pi p+(1-\pi)(1-p)$$

which simplifies to

$$\Pr\left(G|piv^{g}.,g\right)\geq\frac{c}{2p-1}+\pi \tag{1}$$

Since by Bayes' Theorem

$$\Pr\left(G|piv^{g}.,g\right)=\frac{\mu_{G}^{g}p\pi}{\mu_{G}^{g}p\pi+\mu_{B}^{g}(1-\pi)(1-p)}$$

(1) can be written as

$$\frac{\mu_{B}^{g}}{\mu_{G}^{g}}\leq\left[\frac{1}{\frac{c}{2p-1}+\pi}-1\right]\frac{p}{1-p}.\frac{\pi}{1-\pi} \tag{2}$$

Similarly after receiving a signal b , the agent writes review with positive probability if and only if

$$\Pr\left(G|piv^{b}.,b\right)p+\Pr\left(B|piv^{b}.,b\right)(1-p)\leq\pi p+(1-\pi)(1-p)-c \tag{3}$$

Once again

$$\Pr\left(G|piv^{b}.,b\right)=\frac{\mu_{G}^{b}(1-p)\pi}{\mu_{G}^{b}(1-p)\pi+\mu_{B}^{b}p(1-\pi)}$$

and thus (3) becomes

$$\frac{\mu_{B}^{b}}{\mu_{G}^{b}}\geq\left[\frac{1}{\pi-\frac{c}{2p-1}}-1\right]\frac{1-p}{p}.\frac{\pi}{1-\pi} \tag{4}$$

Let x and y be the number of good and bad reviews written by other agents. As already explained an agent with signal g can be pivotal if and only if $x=y-1$. On the other hand an agent with signal b is pivotal if and only if $x=y$. Suppose all the other agents write review with probabilities σ_g and σ_b after signals g and b respectively. We look for an equilibrium in symmetric strategies

In state G, the probability that there are exactly m agents out of n with g signal (the rest have b) is $\binom{n}{m}p^m(1-p)^{n-m}$. Consider any $m \in \{0,1,2,...,n\}$. Since an agent with signal g writes review with probability σ_g, the probability that out of m agents with g signal exactly x agents write reviews is given by $\binom{m}{x}(\sigma_g)^x(1-\sigma_g)^{m-x}$. Similarly the probability that out of $(n-m)$ agents with signal b, exactly y writes review is $\binom{n-m}{y}(\sigma_b)^y(1-\sigma_b)^{n-m-y}$. Hence the probability that an agent with signal g is pivotal in state G is

$$\begin{aligned}\mu_G^g &= \sum_{m=0}^{n}\binom{n}{m}p^m(1-p)^{n-m} \\ &\times \left[\sum_{x=0}^{\min\{m,n-m-1\}}\binom{m}{x}(\sigma_g)^x(1-\sigma_g)^{m-x}\binom{n-m}{x+1}(\sigma_b)^{x+1}(1-\sigma_b)^{n-m-(x+1)}\right]\end{aligned} \tag{5}$$

Notice that neither x can exceed m nor y can exceed $(n-m)$. Since for the agent to be pivotal $y = x+1$, x must be less than both m and $(n-m-1)$. Similarly, the probability that an agent with signal b is pivotal in state G is

$$\begin{aligned}\mu_G^b &= \sum_{m=0}^{n}\binom{n}{m}p^m(1-p)^{n-m} \\ &\times \left[\sum_{x=0}^{\min\{m,n-m\}}\binom{m}{x}(\sigma_g)^x(1-\sigma_g)^{m-x}\binom{n-m}{x}(\sigma_b)^{x}(1-\sigma_b)^{n-m-x}\right]\end{aligned} \tag{6}$$

In state B, the probability of being pivotal after getting signals g and b can similarly be computed as,

$$\begin{aligned}\mu_B^g &= \sum_{m=0}^{n}\binom{n}{m}(1-p)^m p^{n-m} \\ &\times \left[\sum_{x=0}^{\min\{m,n-m-1\}}\binom{m}{x}(\sigma_g)^x(1-\sigma_g)^{m-x}\binom{n-m}{x+1}(\sigma_b)^{x+1}(1-\sigma_b)^{n-m-(x+1)}\right]\end{aligned} \tag{7}$$

and

$$\mu_B^b = \sum_{m=0}^{n} \binom{n}{m} (1-p)^m p^{n-m} \times \left[\sum_{x=0}^{\min\{m,n-m\}} \binom{m}{x} (\sigma_g)^x (1-\sigma_g)^{m-x} \binom{n-m}{x} (\sigma_b)^x (1-\sigma_b)^{n-m-x} \right] \tag{8}$$

We can write the equations in another way for analytical simplicity. For any x since x refers to the number of good reviews and m agents have got good signals $m \geq x$ and $n-m \geq y$. Since upon receiving a good signal the agent is pivotal only when $x = y-1$. Therefore the two conditions become $m \geq x$ and $n-m \geq x+1$.Therefore for any x, we can write the equation for μ_G^g as follows

$$\mu_G^g = \sum_{m=x}^{n-x-1} \binom{n}{m} p^m (1-p)^{n-m} \binom{m}{x} (\sigma_g)^x (1-\sigma_g)^{m-x} \binom{n-m}{x+1} (\sigma_b)^{(x+1)} (1-\sigma_b)^{n-m-(x+1)}$$

By defining a new index $k = m-x$, the equation for μ_G^g can now again be re-written as

$$\begin{aligned} \mu_G^g &= \sum_{k=0}^{n-2x-1} \binom{n}{k+x} p^{k+x} (1-p)^{n-k-x} \\ &\times \left[\binom{k+x}{x} (\sigma_g)^x (1-\sigma_g)^k \binom{n-x-k}{x+1} (\sigma_b)^{(x+1)} (1-\sigma_b)^{n-k-(2x+1)} \right] \\ &= \binom{n}{x} \binom{n-x}{x+1} (p\sigma_g)^x ((1-p)\sigma_b)^{x+1} (1-p\sigma_g-(1-p)\sigma_b)^{n-2x-1} \end{aligned}$$

Now since x runs from 0 to $(n-1)/2$, the equation becomes

$$\mu_G^g = \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x} \binom{n-x}{x+1} (p\sigma_g)^x ((1-p)\sigma_b)^{x+1} (1-p\sigma_g-(1-p)\sigma_b)^{n-2x-1}$$

Similarly , the other equations can be rewritten as

$$\mu_B^g = \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x+1}((1-p)\sigma_g)^x(p\sigma_b)^{x+1}(1-(1-p)\sigma_g-p\sigma_b)^{n-2x-1}$$

$$\mu_G^b = \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x}(p\sigma_g)^x((1-p)\sigma_b)^x(1-p\sigma_g-(1-p)\sigma_b)^{n-2x}$$

$$\mu_B^b = \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x}((1-p)\sigma_g)^x(p\sigma_b)^x(1-(1-p)\sigma_g-p\sigma_b)^{n-2x}$$

3 Results

Proposition 1. *For the conditions*

$$\left[\left[\frac{1}{\pi-\frac{c}{(2p-1)}}-1\right]^{\frac{n-1}{n}}\left(\frac{1-p}{p}\right)^{\frac{n-1}{n}}\left(\frac{\pi}{1-\pi}\right)^{\frac{n-1}{n}} > \left[\frac{1}{\pi+\frac{c}{(2p-1)}}-1\right]\left(\frac{\pi}{1-\pi}\right)\right]$$

and

$$c < \pi(2p-1),$$

we have a mixed strategy equilibrium at $\sigma_g = 0$ *and* $\sigma_b \in [0,1]$

The proof is given in the Appendix

One such parameter conditions where the equilibrium is obtained is $n = 2000, c = .1, \pi = 0.6, p = 0.6$

Since, the number of reviewers in the Internet can become very large, we check to find whether the equilibrium sustains if we increase the number of consumers to infinity. We find that the equilibrium strategy $\sigma_g = 0$ and $\sigma_b \in [0,1]$ holds true even when n tends to infinity.

Proposition 2. *When the condition* $\left[\frac{1}{\pi-\frac{c}{(2p-1)}}-1\right]\left(\frac{1-p}{p}\right) > \left[\frac{1}{\pi+\frac{c}{(2p-1)}}-1\right]$ *holds,* $\sigma_g = 0$ *and* $\sigma_b \in [0,1]$ *is an equilibrium even when* $n \to \infty$

The proof is given in the Appendix

One such parameter value is $\pi = 0.6, c = 0.1, p = 0.9$

Next we try to see when is $\sigma_g = 1$ and $\sigma_b = 1$ an equilibrium

Equation 17 can be written in the following way

$$\frac{c}{2p-1} \leq \frac{1}{\frac{\mu_B^g (1-p)(1-\pi)}{\mu_G^g (p)(\pi)} + 1} - \pi \tag{9}$$

Let the R.H.S of this condition be called R_1

Similarly Equation 18 can be written as

$$\frac{c}{2p-1} \leq \pi - \frac{1}{\frac{\mu_B^b (p)(1-\pi)}{\mu_G^b (1-p)(\pi)} + 1} \tag{10}$$

Let the R.H.S of this condition be called R_2

Now , we can show that

$$\frac{d}{d\sigma_g} \left(\frac{\mu_B^b}{\mu_G^b} \right) > 0 \tag{11}$$

$$\frac{d}{d\sigma_g} \left(\frac{\mu_B^g}{\mu_G^g} \right) > 0 \tag{12}$$

$$\frac{d}{d\sigma_b} \left(\frac{\mu_B^g}{\mu_G^g} \right) < 0 \tag{13}$$

$$\frac{d}{d\sigma_b} \left(\frac{\mu_B^b}{\mu_G^b} \right) < 0 \tag{14}$$

Proof is given in the Appendix

Thus,

R_1 is decreasing in σ_g and increasing in σ_b

R_2 is increasing in σ_g and decreasing in σ_b

This implies that R_1 and the R_2 level curves when drawn for various levels of c, in the σ_g and σ_b plane are positively sloped.

Lemma 1. *At zero cost, the only unique equilibrium is at* $\sigma_g = 1$ *and* $\sigma_b = 1$*.*

The proof is given in the Appendix

In the Figures section of the paper, we show diagrammatically the presence of the (1,1) equilibrium when $c = 0$, and show how the equilibrium shifts as cost increases.In all the diagrams the blue line is the level curve for R_2 and the red line is the level curve for R_1

When $n = 11$, $p = 0.7$, $\pi = 0.8, c = 0\%$, the figure looks like Fig 1. The X- axis denotes σ_g and the Y-axis denotes σ_b

The black arrows show the movement towards the equilibrium from an off equilibrium position. We can illustrate this in the following way

Now (9) can be written as

$$R_1 - \frac{c}{2p-1} \geq 0 \Leftrightarrow \sigma_g = 1 \tag{15}$$

and (10) can be written as

$$R_2 - \frac{c}{2p-1} \geq 0 \Leftrightarrow \sigma_b = 1 \tag{16}$$

It is easy to see from the equations of R_1 and R_2 that R_1 is decreasing in σ_g and increasing in σ_b. R_2 is increasing in σ_g and decreasing in σ_b. Therefore, below the red line σ_b is less than the value that satisfies (15) with equality. As the value of σ_b falls, the value of R_1 too falls, and the (15) gets violated and $\sigma_g \longrightarrow 0$ as shown in the diagram. Below the red line, the value of σ_b is also less than that value of σ_b that satisfies (16). However , as σ_b falls, the value of R_2 rises as R_2 is falling in σ_b. Hence condition (16) gets satisfied more easily and

$\sigma_b \longrightarrow 1$. Thus, the equilibrium force is toward the north-west direction, and will ultimately hit the red line. Once it hits the red line, R_1 is satisfied, but σ_b is still less than the value which satisfies (16) and hence $\sigma_b \longrightarrow 1$. Now, as σ_b increases, it become greater than the value of σ_b which satisfies (15) and hence R_1 increases , and $\sigma_g \longrightarrow 1$. The equilibrium force is therefore north-east direction towards the equilibrium.

On the other hand if we start from a point above the blue line, then σ_b is greater than the values that satisfy the conditions (15) and (16). This means that R_1 rises and hence the (15) is more easily satisfied and hence $\sigma_g \longrightarrow 1$. However as σ_b is greater, R_2 falls and hence (16) is not satisfied , and hence $\sigma_b \longrightarrow 0$.The equilibrium force is therefore in the South west direction, and hits the blue line. As soon as tit hits the blue line (16) is satisfied but $\sigma_g \longrightarrow 1$, and hence enters the zone between the two lines and again moves towards (1,1) as already explained above. Thus (1,1) is the only equilibrium when cost is 0.

Now, we need to show that (1,1) is always within the region marked

Proof. For $\sigma_g = 1$ and $\sigma_b = 1$ at $c = 0$,

$$\lim_{\sigma_g=1,\sigma_g=1} \frac{\mu_B^g}{\mu_G^g} = \frac{p}{1-p}$$

Therefore,

$$\begin{aligned} R_1 &= \frac{1}{1+\frac{1-\pi}{\pi}} - \pi \\ &= 0 \end{aligned}$$

Thus R_1 passes through (1,1) and since R_2 always lies above R_1 we show that at $c = 0$, the only unique equilibrium is at (1,1) □

Lemma 2. *$\sigma_g = 1$ and $\sigma_b = 1$ equilibrium only occurs at $c = 0$*

Proof. As

Now as c increases, the level curve for R_2 shifts downwards because as R_2 increases σ_b falls. Similarly, as c increases, the R_1 curve shifts upwards because as R_1 increases σ_b rises. Therefore at when cost is 1%, the equilibrium is determined in the same manner, which occurs at a point just shy of (1,1) and is a mixed strategy equilibrium. The forces are the same as it was when cost was zero, as the relative position of the curve remains same.

When $n = 11$, $p = 0.7$, $\pi = 0.8, c = 1\%$, the figure looks like the one in 2

As the cost increases to 2%, the gap between the curves diminishes further, but we still get a mixed strategy equilibrium. When $n = 11$, $p = 0.7$, $\pi = 0.8, c = 2\%$, the figure looks like the one showed in Fig 3

When we increase the cost to 3%, the curves have interchanged positions, as shown in Fig 4 and the R_1 curve is above the R_2 curve, which means, the equilibrating forces between the two lines interchange directions and the equilibrium occurs at the point where the R_2 curve cuts the Y-axis such that $\sigma_g = 0$ and σ_b is a positive fraction.

To explain it more clearly, one can see the graphs at cost equal to 4% as the gap between the curves increases further than the one shown in Fig 4, and the equilibrium is obtained at $\sigma_g = 0$ and σ_b is a positive fraction as shown in Fig 5

As the cost increases even more such as to 8%, the R_1 level curve goes out of the graph and hence the equilibrium, goes to $(0, 0)$. The figure looks like the one in Fig 6

Proposition 3. *At zero cost, the equilibrium occurs at $\sigma_g = 1$ and $\sigma_b = 1$. As the cost increases we move towards a mixed strategy equilibrium, and gradually as the cost increases a threshold level, we reach the equilibrium where $\sigma_g = 0$ and $\sigma_b = 1$.*

Thus, we have shown that as the cost of writing review increases , the probability of writing gradually decreases from 1 to 0. However, we have been able to show that consumers do write reviews with a positive probability at a positive cost.

4 Conclusion

In this paper, we have developed a theoretical model to understand the mechanism behind review writing, the need for which was acknowledged in empirical works, and which was missing till now. The need for a theoretical understanding is important to discern the motivation for review writing which can help us better understand if the reviews are complete noise or convey useful information. The knowledge of this fact is also crucial from a policy prescription view, because when reviews actually serve as a signal towards product quality, then stepping up investments in making the Internet accessible to everybody, can help in overcoming the problem of product market information asymmetry not only in the online retail market , but the mechanism could be replicated in other areas too.

Since the online reviews can be accessed by anybody who wants to access it, the reviews are a type of public good, and hence we tried to understand why would consumers contribute to public goods out of their own volition. To solve the problem we have consider two possible scenarios, even in the absence of any exogenous motivation such as rewards or even altruism.

We have considered an adverse selection framework, with rational strategic individuals, who have a positive cost of writing reviews. Applying the pivotal argument for review writing , by strategic consumers, we have identified the equilibrium strategies of the consumers. We have been able to show that at positive costs of review writing, without any extraneous incentives, rational consumers contribute to reviews honestly. Thus reviews indeed serve as a signal towards product quality and can be sustained endogenously even without any extraneous benefits. As a part of the future work, we would want to model the producer's decision in response to the reviews and analyze both sides of the problem. Another area that we want to look into is to relax the assumption of repeat purchase and explore the results obtained. Last but not the least we want to be able test the theory either experimentally or empirically, whichever suits us the most.

References

Cabral, L. and A. Hortacsu (2010). The dynamics of seller reputation: Evidence from ebay. *The Journal of Industrial Economics 58*(1), 54–78.

Dellarocas, C., G. Gao, and R. Narayan (2010). Are consumers more likely to contribute online reviews for hit or niche products? *Journal of Management Information Systems 27*(2), 127–158.

Dixit, A. (2003). Trade expansion and contract enforcement. *Journal of Political Economy 111*(6), 1293–1317.

Duan, W., B. Gu, and A. B. Whinston (2008). Do online reviews matter?—an empirical investigation of panel data. *Decision support systems 45*(4), 1007–1016.

Feddersen, T. and W. Pesendorfer (1997). Voting behavior and information aggregation in elections with private information. *Econometrica: Journal of the Econometric Society*, 1029–1058.

Feddersen, T. J. and W. Pesendorfer (1996). The swing voter's curse. *The American economic review*, 408–424.

Greif, A. (1993). Contract enforceability and economic institutions in early trade: The maghribi traders' coalition. *The American economic review*, 525–548.

Hu, N., L. Liu, and V. Sambamurthy (2011). Fraud detection in online consumer reviews. *Decision Support Systems 50*(3), 614–626.

Jøsang, A., R. Ismail, and C. Boyd (2007). A survey of trust and reputation systems for online service provision. *Decision support systems 43*(2), 618–644.

Kandori, M. (1992). Social norms and community enforcement. *The Review of Economic Studies 59*(1), 63–80.

Lee, J., D.-H. Park, and I. Han (2008). The effect of negative online consumer reviews on product attitude: An information processing view. *Electronic commerce research and applications 7*(3), 341–352.

Milgrom, P. R., D. C. North, and B. R. Weingast* (1990). The role of institutions in the revival of trade: The law merchant, private judges, and the champagne fairs. *Economics & Politics 2*(1), 1–23.

Resnick, P. and R. Zeckhauser (2002). Trust among strangers in internet transactions: Empirical analysis of ebay's reputation system. In *The Economics of the Internet and E-commerce*, pp. 127–157. Emerald Group Publishing Limited.

5 Figures

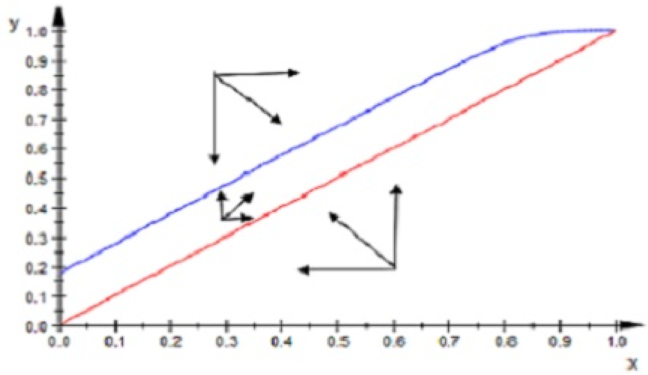

Figure 1: Equilibrium at cost zero

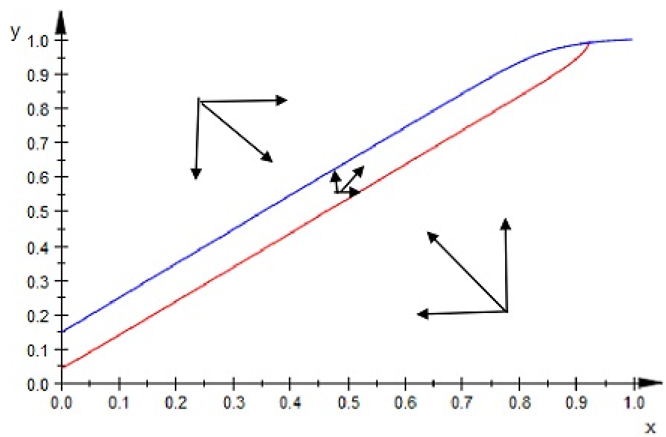

Figure 2: Equilibrium at one percent cost

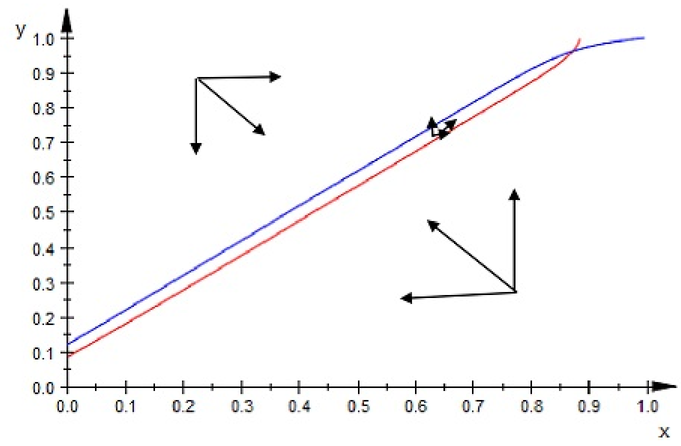

Figure 3: Equilibrium at two percent cost

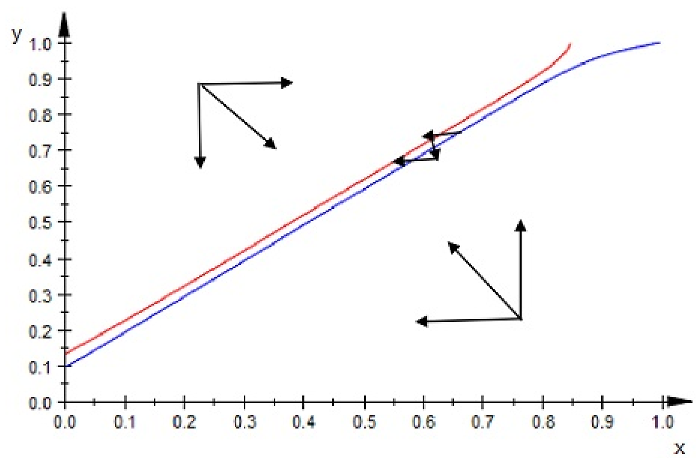

Figure 4: Equilibrium at three percent cost

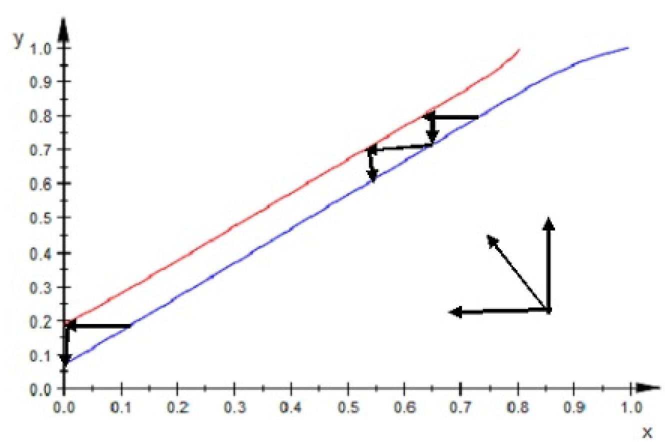

Figure 5: Equilibrium at four percent cost

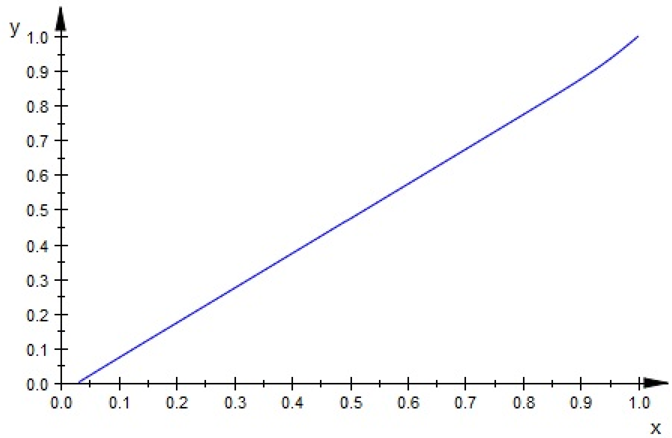

Figure 6: Equilibrium at eight percent cost

6 Appendix

6.1 Proof of Proposition 1

For the conditions

$$\left[\left[\frac{1}{\pi - \frac{c}{(2p-1)}} - 1\right]^{\frac{n-1}{n}} \left(\frac{1-p}{p}\right)^{\frac{n-1}{n}} \left(\frac{\pi}{1-\pi}\right)^{\frac{n-1}{n}} > \left[\frac{1}{\pi + \frac{c}{(2p-1)}} - 1\right] \left(\frac{\pi}{1-\pi}\right)\right]$$

and

$$c < \pi(2p-1),$$

we have a mixed strategy equilibrium at $\sigma_g = 0$ and $\sigma_b \in [0,1]$

Proof. Equilibrium in the following strategy $\sigma_g = 0, \sigma_b = [0,1]$

The conditions for this sort of an equilibrium to hold are as follows

$$\frac{\mu_B^g}{\mu_G^g} > \left[\frac{1}{\frac{c}{2p-1} + \pi} - 1\right] \frac{p}{1-p} . \frac{\pi}{1-\pi} \tag{17}$$

$$\frac{\mu_B^b}{\mu_G^b} = \left[\frac{1}{\pi - \frac{c}{2p-1}} - 1\right] \frac{1-p}{p} . \frac{\pi}{1-\pi} \tag{18}$$

Evaluating $\frac{\mu_B^b}{\mu_G^b}$, at $\sigma_g = 0$, all the terms vanish except for $x = 0$. Thus substituting the value of σ_g in $\frac{\mu_B^g}{\mu_G^g}$ and $\frac{\mu_B^b}{\mu_G^b}$, we get

$$\frac{\mu_B^b}{\mu_G^b} = \frac{(1 - p\sigma_b)^n}{(1 - (1 - p)\,\sigma_b)^n}$$

and

$$\frac{\mu_B^g}{\mu_G^g} = \frac{p(1 - p\sigma_b)^{n-1}}{(1 - p)(1 - (1 - p)\sigma_b)^{n-1}}$$

Now according to the condition 18

$$\frac{(1 - p\sigma_b)^n}{(1 - (1 - p)\,\sigma_b)^n} = \left[\frac{1}{\pi - \frac{c}{2p-1}} - 1\right]\frac{1 - p}{p}.\frac{\pi}{1 - \pi} \tag{19}$$

By substituting the simplified expression for $\frac{\mu_B^g}{\mu_G^g}$ in 17

$$\frac{p(1 - p\sigma_b)^{n-1}}{(1 - p)(1 - (1 - p)\sigma_b)^{n-1}} > \left[\frac{1}{\frac{c}{2p-1} + \pi} - 1\right]\frac{p}{1 - p}.\frac{\pi}{1 - \pi} \tag{20}$$

Now substituting the value of

$$\frac{(1 - p\sigma_b)^n}{(1 - (1 - p)\,\sigma_b)^n}$$

from condition 19 in condition 20

we have

$$\left[\frac{1}{\pi - \frac{c}{(2p-1)}} - 1\right]^{\frac{n-1}{n}}\left(\frac{1 - p}{p}\right)^{\frac{n-1}{n}}\left(\frac{\pi}{1 - \pi}\right)^{\frac{n-1}{n}} > \left[\frac{1}{\pi + \frac{c}{(2p-1)}} - 1\right]\left(\frac{\pi}{1 - \pi}\right) \tag{21}$$

This gives us one restriction on the value of c for the other parameters, another restriction is that

$$c < \pi(2p-1) \qquad \square$$

6.2 Proof of Proposition 2

For the conditions

$$\left[\left[\frac{1}{\pi - \frac{c}{(2p-1)}} - 1\right]^{\frac{n-1}{n}} \left(\frac{1-p}{p}\right)^{\frac{n-1}{n}} \left(\frac{\pi}{1-\pi}\right)^{\frac{n-1}{n}} > \left[\frac{1}{\pi + \frac{c}{(2p-1)}} - 1\right] \left(\frac{\pi}{1-\pi}\right)\right]$$

and

$$c < \pi(2p-1),$$

we have a mixed strategy equilibrium at $\sigma_g = 0$ and $\sigma_b \in [0,1]$ when $n \to \infty$

Proof.

$$\left[\frac{1}{\pi - \frac{c}{(2p-1)}} - 1\right] \left(\frac{1-p}{p}\right) > \left[\frac{1}{\pi + \frac{c}{(2p-1)}} - 1\right]$$

This condition, holds for some values of p, π and c which means that even when the number of consumers tends to infinity, as is generally the case in the commodity market, the equilibrium is sustained. $\square$

6.3 Proof of Equations 11, 12, 13 and 14

Proof.

$$\frac{d}{d\sigma_g}\left(\frac{\mu_B^b}{\mu_G^b}\right) > 0$$

$$\frac{d}{d\sigma_g}\left(\frac{\mu_B^b}{\mu_G^b}\right) = \frac{\mu_G^b \frac{d\mu_B^b}{d\sigma_g} - \mu_B^b \frac{d\mu_G^b}{d\sigma_g}}{(\mu_G^b)^2}$$

$$\mu_B^b = \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x} \binom{n-x}{x} p^x (1-p)^x \sigma_g^x \sigma_b^x (1 - (1-p)\sigma_g - p\sigma_b)^{n-2x}$$

$$\frac{d\mu_B^b}{d\sigma_g} = \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x} p^x(1-p)^x x\sigma_g^{(x-1)}\sigma_b^x(1-(1-p)\sigma_g - p\sigma_b)^{n-2x} - \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x} p^x(1-p)^x \sigma_g^x\sigma_b^x(1-p)(n-2x)(1-(1-p)\sigma_g - p\sigma_b)^{n-2x-1}$$

$$\mu_G^b = \sum_{y=0}^{\frac{n-1}{2}} \binom{n}{y}\binom{n-y}{y} p^y(1-p)^y \sigma_g^y\sigma_b^y(1-p\sigma_g-(1-p)\sigma_b)^{n-2y}$$

$$\frac{d\mu_G^b}{d\sigma_g} = \sum_{y=0}^{\frac{n-1}{2}} \binom{n}{y}\binom{n-y}{y} p^y(1-p)^y y\sigma_g^{(y-1)}\sigma_b^y(1-(1-p)\sigma_b - p\sigma_g)^{n-2y} - \sum_{y=0}^{\frac{n-1}{2}} \binom{n}{y}\binom{n-y}{y} p^y(1-p)^y \sigma_g^y\sigma_b^y(p)(n-2y)(1-(1-p)\sigma_b - p\sigma_g)^{n-2y-1}$$

Multiplying the first terms of the differential with we get

$$\begin{aligned}\frac{d}{d\sigma_g}\left(\frac{\mu_B^b}{\mu_G^b}\right)\times(\mu_G^b)^2 = {} & (\sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x}\binom{n}{y}\binom{n-y}{y} p^{(x+y)}(1-p)^{(x+y)} x\sigma_g^{(x+y-1)}\sigma_b^{y+x} \\ & \times[(1-(1-p)\sigma_g-p\sigma_b)^{n-2x}(1-p\sigma_g-(1-p)\sigma_b)^{n-2y}]) - \\ & (\sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x}\binom{n}{y}\binom{n-y}{y} p^{(x+y)}(1-p)^{(x+y)} y\sigma_g^{(x+y-1)}\sigma_b^{y+x} \\ & \times[(1-(1-p)\sigma_g-p\sigma_b)^{n-2x}(1-p\sigma_g-(1-p)\sigma_b)^{n-2y}]) + \\ & (\sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x}\binom{n}{y}\binom{n-y}{y} p^{(x+y+1)}(1-p)^{(x+y)} \sigma_g^{(x+y-1)}\sigma_b^{y+x} \\ & \times[(n-2y)(1-(1-p)\sigma_g-p\sigma_b)^{n-2x}(1-p\sigma_g-(1-p)\sigma_b)^{n-2y-1}]) - \\ & (\sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x}\binom{n}{y}\binom{n-y}{y} p^{(x+y)}(1-p)^{(x+y+1)} \sigma_g^{(x+y-1)}\sigma_b^{y+x} \\ & [(n-2x)(1-(1-p)\sigma_g-p\sigma_b)^{n-2x-1}(1-p\sigma_g-(1-p)\sigma_b)^{n-2y}])\end{aligned}$$

This first two terms cancel out and, and the equation becomes

$$\begin{aligned}\frac{d}{d\sigma_g}\left(\frac{\mu_B^b}{\mu_G^b}\right) &= \sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}}\binom{n}{x}\binom{n-x}{x}\binom{n}{y}\binom{n-y}{y}p^{(x+y)}(1-p)^{(x+y)}\sigma_g^{(x+y-1)}\sigma_b^{y+x}(n-2y)\\ &\quad\times(n-2x)(1-(1-p)\sigma_g-p\sigma_b)^{n-2x-1}(1-p\sigma_g-(1-p)\sigma_b)^{n-2y-1}\\ &\quad\times[p(1-(1-p)\sigma_b-p\sigma_g)^{-1}-(1-p)(1-p\sigma_b-(1-p)\sigma_g)^{-1}]\end{aligned}$$

Thus, if the bracketed portion is positive or negative then, the entire expression is positive or negative depending upon that

Thus the entire expression is greater than or less than zero depending upon whether

$$[p(1-(1-p)\sigma_b-p\sigma_g)^{-1}-(1-p)(1-p\sigma_b-(1-p)\sigma_g)^{-1}] \lesseqgtr 0$$

Evaluating

$$\begin{aligned}&[p(1-(1-p)\sigma_b-p\sigma_g)^{-1}-(1-p)(1-p\sigma_b-(1-p)\sigma_g)^{-1}]\\ =\ &(2p-1)(1-\sigma_b)\end{aligned}$$

This is always greater than zero.

Therefore .

$$\frac{d}{d\sigma_g}\left(\frac{\mu_B^b}{\mu_G^b}\right) > 0$$

□

The second proof is as follows

Proof.

$$\frac{d}{d\sigma_g}\left(\frac{\mu_B^g}{\mu_G^g}\right) > 0$$

$$\frac{d}{d\sigma_g}\left(\frac{\mu_B^g}{\mu_G^g}\right) = \frac{\mu_G^g \frac{d\mu_B^g}{d\sigma_g} - \mu_B^g \frac{d\mu_G^g}{d\sigma_g}}{(\mu_G^g)^2}$$

$$\mu_G^g = \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x+1} p^x (1-p)^{x+1} \sigma_g^x \sigma_b^{x+1} (1 - p\sigma_g - (1-p)\sigma_b)^{n-2x-1}$$

$$\begin{aligned}\frac{d\mu_G^g}{d\sigma_g} \quad = \quad & \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x+1} p^x (1-p)^{x+1} x \sigma_g^{(x-1)} \sigma_b^{x+1} (1 - p\sigma_g - (1-p)\sigma_b)^{n-2x-1} - \\ & \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x+1} p^x (1-p)^{x+1} \sigma_g^x \sigma_b^{x+1} p(n-2x-1)(1 - p\sigma_g - (1-p)\sigma_b)^{n-2x-2}\end{aligned}$$

$$\mu_B^g = \sum_{y=0}^{\frac{n-1}{2}} \binom{n}{y}\binom{n-y}{y+1} p^{y+1} (1-p)^{y} \sigma_g^y \sigma_b^{y+1} (1 - (1-p)\sigma_g - p\sigma_b)^{n-2y-1}$$

$$\begin{aligned}\frac{d\mu_B^g}{d\sigma_g} \quad = \quad & \sum_{y=0}^{\frac{n-1}{2}} \binom{n}{y}\binom{n-y}{y+1} p^{y+1} (1-p)^{y} y \sigma_g^{(y-1)} \sigma_b^{y+1} (1 - (1-p)\sigma_g - p\sigma_b)^{n-2y-1} - \\ & \sum_{y=0}^{\frac{n-1}{2}} \binom{n}{y}\binom{n-y}{y+1} p^{y+1} (1-p)^{y} \sigma_g^y \sigma_b^{y+1} (1-p)(n-2y-1)(1 - (1-p)\sigma_g - p\sigma_b)^{n-2y-2}\end{aligned}$$

Multiplying the first terms of the differential with we get

$$
\begin{aligned}
\frac{d}{d\sigma_g}\left(\frac{\mu_B^g}{\mu_G^g}\right)\times(\mu_G^g)^2 \;=\; & (\sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}}\binom{n}{x}\binom{n-x}{x+1}\binom{n}{y}\binom{n-y}{y+1}p^{x+y+1}(1-p)^{x+y+1}y\sigma_g^{x+y-1}\sigma_b^{y+x+2} \\
& \times[(1-(1-p)\sigma_g-p\sigma_b)^{n-2y-1}(1-p\sigma_g-(1-p)\sigma_b)^{n-2x-1}]) - \\
& (\sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}}\binom{n}{x}\binom{n-x}{x+1}\binom{n}{y}\binom{n-y}{y+1}p^{x+y+1}(1-p)^{x+y+1}x\sigma_g^{x+y-1}\sigma_b^{y+x+2} \\
& \times[(1-(1-p)\sigma_g-p\sigma_b)^{n-2y-1}(1-p\sigma_g-(1-p)\sigma_b)^{n-2x-1}]) + \\
& (\sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}}\binom{n}{x}\binom{n-x}{x+1}\binom{n}{y}\binom{n-y}{y+1}p^{x+y+2}(1-p)^{x+y+1}\sigma_g^{(x+y)}\sigma_b^{y+x+2} \\
& \times[(n-2x-1)(1-(1-p)\sigma_g-p\sigma_b)^{n-2y-1} \\
& \times(1-p\sigma_g-(1-p)\sigma_b)^{n-2x-2}]) - \\
& (\sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}}\binom{n}{x}\binom{n-x}{x+1}\binom{n}{y}\binom{n-y}{y+1}p^{x+y+1}(1-p)^{x+y+2}\sigma_g^{x+y-1}\sigma_b^{y+x+2} \\
& [(n-2y-1)(1-(1-p)\sigma_g-p\sigma_b)^{n-2x-2}(1-p\sigma_g-(1-p)\sigma_b)^{n-2y-1}])
\end{aligned}
$$

This first two terms cancel out and, and the equation becomes

$$
\begin{aligned}
\frac{d}{d\sigma_g}\left(\frac{\mu_B^b}{\mu_G^b}\right) \;=\; & \sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}}\binom{n}{x}\binom{n-x}{x+1}\binom{n}{y}\binom{n-y}{y+1}p^{x+y+1}(1-p)^{x+y+1}\sigma_g^{x+y}\sigma_b^{y+x+2}(n-2x-1) \\
& \times(n-2y-1)(1-(1-p)\sigma_g-p\sigma_b)^{n-2x-1}(1-p\sigma_g-(1-p)\sigma_b)^{n-2y-1} \\
& \times[p(1-(1-p)\sigma_b-p\sigma_g)^{-1}-(1-p)(1-p\sigma_b-(1-p)\sigma_g)^{-1}]
\end{aligned}
$$

Thus, if the bracketed portion is positive or negative then, the entire expression is positive or negative depending upon that

Thus the entire expression is greater than or less than zero depending upon whether

$$[p(1-(1-p)\sigma_b-p\sigma_g)^{-1}-(1-p)(1-p\sigma_b-(1-p)\sigma_g)^{-1}]\lesseqgtr 0$$

Evaluating

$$\begin{aligned} & [p(1-(1-p)\sigma_b - p\sigma_g)^{-1} - (1-p)(1-p\sigma_b-(1-p)\sigma_g)^{-1}] \\ = \; & (2p-1)(1-\sigma_b) \end{aligned}$$

This is always greater than zero.

Therefore .

$$\frac{d}{d\sigma_g}\left(\frac{\mu_B^g}{\mu_G^g}\right) > 0$$

□

The third proof is as follows

Proof.

$$\frac{d}{d\sigma_b}\left(\frac{\mu_B^g}{\mu_G^g}\right) < 0$$

$$\frac{d}{d\sigma_b}\left(\frac{\mu_B^g}{\mu_G^g}\right) = \frac{\mu_G^g \frac{d\mu_B^g}{d\sigma_b} - \mu_B^g \frac{d\mu_G^g}{d\sigma_b}}{(\mu_G^g)^2}$$

$$\mu_G^g = \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x+1} p^x(1-p)^{x+1}\sigma_g^x\sigma_b^{x+1}(1-p\sigma_g-(1-p)\sigma_b)^{n-2x-1}$$

$$\begin{aligned} \frac{d\mu_G^g}{d\sigma_b} = \; & \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x+1} p^x(1-p)^{x+1}(x+1)\sigma_g^x\sigma_b^x(1-p\sigma_g-(1-p)\sigma_b)^{n-2x-1} - \\ & \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x+1} p^x(1-p)^{x+1}\sigma_g^x\sigma_b^{x+1}(1-p)(n-2x-1)(1-p\sigma_g-(1-p)\sigma_b)^{n-2x-2} \end{aligned}$$

$$\mu_B^g = \sum_{y=0}^{\frac{n-1}{2}} \binom{n}{y}\binom{n-y}{y+1} p^{y+1}(1-p)^y\sigma_g^y\sigma_b^{y+1}(1-(1-p)\sigma_g-p\sigma_b)^{n-2y-1}$$

$$\begin{aligned}\frac{d\mu_B^g}{d\sigma_b} &= \sum_{y=0}^{\frac{n-1}{2}} \binom{n}{y}\binom{n-y}{y+1} p^{y+1}(1-p)^y \sigma_g^y (y+1)\sigma_b^y (1-(1-p)\sigma_g - p\sigma_b)^{n-2y-1} - \\ & \sum_{y=0}^{\frac{n-1}{2}} \binom{n}{y}\binom{n-y}{y+1} p^{y+1}(1-p)^y \sigma_g^y \sigma_b^{y+1} p(n-2y-1)(1-(1-p)\sigma_g - p\sigma_b)^{n-2y-2}\end{aligned}$$

Multiplying the terms of the differential with we get

$$\begin{aligned}\frac{d}{d\sigma_b}\left(\frac{\mu_B^g}{\mu_G^g}\right) \times (\mu_G^g)^2 &= (\sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x+1}\binom{n}{y}\binom{n-y}{y+1}(p(1-p)^{x+y+1}(y+1)\sigma_g^{x+y}\sigma_b^{y+x+1} \\ & \times[(1-(1-p)\sigma_g - p\sigma_b)^{n-2y-1}(1-p\sigma_g-(1-p)\sigma_b)^{n-2x-1}]) - \\ & (\sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x+1}\binom{n}{y}\binom{n-y}{y+1}(p(1-p))^{x+y+1}(x+1)\sigma_g^{x+y}\sigma_b^{y+x+1} \\ & \times[(1-(1-p)\sigma_g - p\sigma_b)^{n-2x-1}(1-p\sigma_g-(1-p)\sigma_b)^{n-2y-1}]) + \\ & (\sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x+1}\binom{n}{y}\binom{n-y}{y+1}p^{x+y+1}(1-p)^{x+y+2}\sigma_g^{x+y}\sigma_b^{y+x+2} \\ & \times[(n-2x-1)(1-(1-p)\sigma_g - p\sigma_b)^{n-2y-1} \\ & \times(1-p\sigma_g-(1-p)\sigma_b)^{n-2x-2}]) - \\ & (\sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x+1}\binom{n}{y}\binom{n-y}{y+1}p^{x+y+2}(1-p)^{x+y+1}\sigma_g^{x+y}\sigma_b^{y+x+2} \\ & [(n-2y-1)p(1-(1-p)\sigma_g - p\sigma_b)^{n-2y-2}(1-p\sigma_g-(1-p)\sigma_b)^{n-2x-1}])\end{aligned}$$

This first two terms cancel out and, and the equation becomes

$$\begin{aligned}\frac{d}{d\sigma_b}\left(\frac{\mu_B^b}{\mu_G^b}\right) &= \sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x+1}\binom{n}{y}\binom{n-y}{y+1}p^{x+y+1}(1-p)^{x+y+1}\sigma_g^{x+y}\sigma_b^{y+x+2}(n-2x-1) \\ & \times(n-2y-1)(1-(1-p)\sigma_g - p\sigma_b)^{n-2x-1}(1-p\sigma_g-(1-p)\sigma_b)^{n-2y-1} \\ & \times[(1-p)(1-(1-p)\sigma_b - p\sigma_g)^{-1} - p(1-p\sigma_b-(1-p)\sigma_g)^{-1}]\end{aligned}$$

Thus, if the bracketed portion is positive or negative then, the entire expression is positive or negative depending upon that

Thus the entire expression is greater than or less than zero depending upon whether

$$[(1-p)(1-(1-p)\sigma_b - p\sigma_g)^{-1} - p(1-p\sigma_b-(1-p)\sigma_g)^{-1}] \lesseqgtr 0$$

Evaluating

$$\begin{aligned} & [(1-p)(1-(1-p)\sigma_b - p\sigma_g)^{-1} - p(1-p\sigma_b-(1-p)\sigma_g)^{-1}] \\ = \; & (2p-1)(\sigma_g - 1) \end{aligned}$$

This is always less than zero. □

Proof.

$$\frac{d}{d\sigma_b}\left(\frac{\mu_B^b}{\mu_G^b}\right) > 0$$

$$\frac{d}{d\sigma_b}\left(\frac{\mu_B^b}{\mu_G^b}\right) = \frac{\mu_G^b \frac{d\mu_B^b}{d\sigma_b} - \mu_B^b \frac{d\mu_G^b}{d\sigma_b}}{(\mu_G^b)^2}$$

$$\mu_B^b = \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x} p^x (1-p)^x \sigma_g^x \sigma_b^x (1-(1-p)\sigma_g - p\sigma_b)^{n-2x}$$

$$\begin{aligned} \frac{d\mu_B^b}{d\sigma_b} = \; & \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x} p^x (1-p)^x \sigma_g^{x-} x \sigma_b^{(x-1)} (1-(1-p)\sigma_g - p\sigma_b)^{n-2x} - \\ & \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x} p^x (1-p)^x \sigma_g^x \sigma_b^x p(n-2x)(1-(1-p)\sigma_g - p\sigma_b)^{n-2x-1} \end{aligned}$$

$$\mu_G^b = \sum_{y=0}^{\frac{n-1}{2}} \binom{n}{y}\binom{n-y}{y} p^y (1-p)^y \sigma_g^y \sigma_b^y (1-p\sigma_g - (1-p)\sigma_b)^{n-2y}$$

$$\frac{d\mu_G^b}{d\sigma_g} = \sum_{y=0}^{\frac{n-1}{2}} \binom{n}{y}\binom{n-y}{y} p^y(1-p)^y \sigma_g^y y \sigma_b^{(y-1)}(1-(1-p)\sigma_b - p\sigma_g)^{n-2y} - \sum_{y=0}^{\frac{n-1}{2}} \binom{n}{y}\binom{n-y}{y} p^y(1-p)^y \sigma_g^y \sigma_b^y (1-p)(n-2y)(1-(1-p)\sigma_b - p\sigma_g)^{n-2y-1}$$

Multiplying the first terms of the differential with we get

$$\begin{aligned}
\frac{d}{d\sigma_g}\left(\frac{\mu_B^b}{\mu_G^b}\right) \times (\mu_G^b)^2 = {} & (\sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x}\binom{n}{y}\binom{n-y}{y} p^{(x+y)}(1-p)^{(x+y)}\sigma_g^{(x+y)} x \sigma_b^{y+x-1} \\
& \times[(1-(1-p)\sigma_g - p\sigma_b)^{n-2x}(1-p\sigma_g-(1-p)\sigma_b)^{n-2y}]) - \\
& (\sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x}\binom{n}{y}\binom{n-y}{y} p^{(x+y)}(1-p)^{(x+y)}\sigma_g^{(x+y)} y \sigma_b^{y+x-1} \\
& \times[(1-(1-p)\sigma_g - p\sigma_b)^{n-2x}(1-p\sigma_g-(1-p)\sigma_b)^{n-2y}]) + \\
& (\sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x}\binom{n}{y}\binom{n-y}{y} p^{(x+y)}(1-p)^{(x+y+1)}\sigma_g^{(x+y)} \sigma_b^{y+x-1} \\
& \times[(n-2y)(1-(1-p)\sigma_g - p\sigma_b)^{n-2x}(1-p\sigma_g-(1-p)\sigma_b)^{n-2y-1}]) - \\
& (\sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x}\binom{n}{y}\binom{n-y}{y} p^{(x+y+1)}(1-p)^{(x+y)}\sigma_g^{(x+y)} \sigma_b^{y+x} \\
& [(n-2x)(1-(1-p)\sigma_g - p\sigma_b)^{n-2x-1}(1-p\sigma_g-(1-p)\sigma_b)^{n-2y}])
\end{aligned}$$

This first two terms cancel out and, and the equation becomes

$$\begin{aligned}
\frac{d}{d\sigma_b}\left(\frac{\mu_B^b}{\mu_G^b}\right) = {} & \sum_{x=0}^{\frac{n-1}{2}}\sum_{y=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x}\binom{n}{y}\binom{n-y}{y} p^{(x+y)}(1-p)^{(x+y)}\sigma_g^{(x+y-1)}\sigma_b^{y+x}(n-2y) \\
& \times(n-2x)(1-(1-p)\sigma_g - p\sigma_b)^{n-2x-1}(1-p\sigma_g-(1-p)\sigma_b)^{n-2y-1} \\
& \times[(1-p)(1-p\sigma_g-(1-p)\sigma_b)^{-1} - p(1-(1-p)\sigma_g-p\sigma_b)^{-1}]
\end{aligned}$$

Thus, if the bracketed portion is positive or negative then, the entire expression is positive or negative depending upon that

Thus the entire expression is greater than or less than zero depending upon whether

$$[(1-p)(1-p\sigma_g-(1-p)\sigma_b)^{-1}-p(1-(1-p)\sigma_g-p\sigma_b)^{-1}] \lesseqgtr 0$$

Evaluating

$$lbrack(1-p)(1-p\sigma_g-(1-p)\sigma_b)^{-1}-p(1-(1-p)\sigma_g-p\sigma_b)^{-1}]] \\ = (2p-1)(\sigma_g-1)$$

This is always less than zero. □

6.4 Proof of Lemma 1

At zero cost, the only unique equilibrium is at $\sigma_g = 1$ and $\sigma_b = 1$.

Proof. R_2 curve always lies above the R_1 for zero cost for all values of p, π

For this, we need to be able to show that for $c = 0$ the values of σ_g and σ_b at which the R_1equation holds with equality, for any particular σ_g, the value of σ_b at which R_2 holds with equality should be greater and hence the R_2 curve will lie above the R_1 curve.

For R_1 to hold with equality

$$\frac{c}{2p-1} = \frac{1}{\frac{\mu_B^g(1-p)(1-\pi)}{\mu_G^g(p)(\pi)}+1} - \pi$$

$$\implies \frac{\mu_B^g}{\mu_G^g} = \frac{p}{1-p}$$

Putting in the values of μ_B^b and μ_G^b, we get

$$\sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x+1}(1-p)^x\sigma_g^x p^x \sigma_b^{(x+1)}(1-(1-p)\sigma_g - p\sigma_b)^{n-2x-1}$$
$$= \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x+1}(1-p)^x\sigma_g^x p^x \sigma_b^{(x+1)}(1-p\sigma_g-(1-p)\sigma_b)^{n-2x-1} \quad (22)$$

Now for R_2 to lie above the R_1 curve, we need to show that the level curve for

$$\frac{c}{2p-1} = \pi - \frac{1}{\frac{\mu_B^b(p)(1-\pi)}{\mu_G^b(1-p)(\pi)}+1}$$

Putting $c = 0$, we get

$$\frac{\mu_B^b}{\mu_G^b} = \frac{1-p}{p}$$

Evaluating this we get

$$\sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x+1}(1-p)^x\sigma_g^x p^x \sigma_b^{(x+1)}(1-(1-p)\sigma_g - p\sigma_b)^{n-2x-1}\left[\frac{p}{\sigma_b}(1-(1-p)\sigma_g-p\sigma_b)\right]$$
$$= \sum_{x=0}^{\frac{n-1}{2}} \binom{n}{x}\binom{n-x}{x+1}(1-p)^x\sigma_g^x p^x \sigma_b^{(x+1)}(1-p\sigma_g-(1-p)\sigma_b)^{n-2x-1}$$
$$\times\left[\frac{(1-p)}{\sigma_b}(1-p\sigma_g-(1-p)\sigma_b)\right]$$

We can see that except for the

$$\left[\frac{p}{\sigma_b}(1-(1-p)\sigma_g-p\sigma_b)\right] \quad (23)$$

and the

$$\left[\frac{(1-p)}{\sigma_b}(1-p\sigma_g-(1-p)\sigma_b)\right] \quad (24)$$

the rest of the equation is identical to the one in condition **??**, hence if we can show that condition 23 is greater than 24, for each of the x, then the R_2 curve is above the R_1 curve. Thus, we have to show that

$$\left[\frac{p}{\sigma_b}\left(1-(1-p)\sigma_g - p\sigma_b\right)\right] \quad > \quad \left[\frac{(1-p)}{\sigma_b}\left(1-p\sigma_g-(1-p)\sigma_b\right)\right]$$
$$\implies [2p-1] > [(2p-1)\sigma_b]$$

which holds true for all values of σ_b. □

Thus, we have been able to prove that at zero cost for all values of σ_g and σ_b, R_2 lies above the R_1 curve.